Large-Eddy and Wall-Modeled Simulations of Turbulent Flow over Two-Dimensional River Dunes

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Abstract

In this research, models of the Detached-Eddy Simulation (DES) family [2, 3, 4] are used for calculations of turbulent flow over a two-dimensional dune geometry at $Re_b = U_b H_b / \nu = 50\,000$. This case is neither a fully attached flow nor a massively separated flow and seems to be a good candidate to investigate the cost and accuracy of DES wall-models in such cases. Results are compared to the well-validated LES database. Compared to attached, equilibrium flows or flows with a mild separation, a higher accuracy was achieved in prediction of the mean and second order statistics using all the present hybrid methods. All the methods saved up to 97% of the total computational cost (compared to the LES simulations based on the required CPU time). Moreover, considering both the first and second order statistics, as well as the flow physics included in the instantaneous turbulent field of the simulation, the IDDES model resulted in a higher level of accuracy compared to the DES and DDES methods. However, the same grid density as the two other models might lead to generation of small-scale uncorrelated structures over the reattachment region. To resolve this issue it might be necessary to refine the mesh in wall-parallel directions (streamwise and spanwise directions) when using IDDES as the wall-layer model.

1 Introduction

The Large-Eddy Simulation technique (LES) suffers from grid resolution requirement when a solid wall is present in domain of a simulation, same as Direct Numerical Simulation (DNS), which makes that infeasible for realistic, high Reynolds number applications. To resolve this issue, one can bypass the near-wall region and model its effects on the outer flow in a statistical sense; that can result in saving of more than 90% of the grid points expended to resolve less than 10% of the total volume in the near-wall area [6]. This approach is known as Wall-Modeled LES (WMLES) and extends the capabilities of large-eddy simulation technique to high Reynolds number wall-bounded flows. In this regard, the hybrid Reynolds-Averaged Navier-Stokes (RANS)/LES techniques blend a RANS-type turbulent eddy-viscosity near the wall and a LES subgrid eddy-viscosity in the rest of the domain and solve the filtered equations on a single mesh, resulting in a strong coupling between the inner and outer-layer flow.

A unified hybrid RANS/LES model should have the following properties: (i) result in a RANS (or URANS when a separation of scales is present in the flow) solution when applied along with a coarse mesh; (ii) the effect of explicit filtering on the model should be negligible (a usual consequence of numerical discretization as well as the order of accuracy of the numerical integration); (iii) the subgrid-scale model should automatically turn off if the grid resolution is of the order of DNS; (iv) statistical and instantaneous features of the solution should be recovered, independent of the initial condition used for the simulation [7].

The hybrid simulation strategy was firstly prompted by the required computational power for LES of airplane wings at Reynolds numbers near flight values. Spalart & Allmaras [2] proposed the DES model that reduces to RANS-like solution in the attached boundary layers and LES after separation. The new formulation modifies the turbulence length scale d (distance to the nearest wall) in the base RANS eddy-viscosity equation as follows,

$$\tilde{d} = \min(d, C_{DES}\Delta), \quad \Delta = \max(\Delta x, \Delta y, \Delta z) \tag{1}$$

where $C_{\text{DES}} = 0.65$ is the model constant and Δx, Δy, and Δz are the grid spacings in wall-parallel and wall-normal directions. This approach was originally intended for massively separated flows where the instability of the shear layer after separation is strong enough to accelerate the eddy-generation at the RANS/LES interface. Therefore, the problem associated with the "grey area" is no more present (the grey area of the hybrid RANS/LES techniques is the region where the model switches between RANS and LES behaviors. There, the Reynolds shear stress is usually under-predicted; this can lead to the log-layer mismatch or under-prediction of the wall shear stress).

The Sparalrt-Allmaras RANS eddy-viscosity equation (hereinafter referred to as SA-RANS [1]) is given by the following formula, equation 2.

$$\underbrace{\frac{D\tilde{\nu}}{Dt}}_{\text{advection}} = \underbrace{c_{b1}\tilde{S}\tilde{\nu}}_{\text{production}} \underbrace{-c_{w1} f_w [\frac{\tilde{\nu}}{d}]^2}_{\text{destruction}} + \underbrace{\frac{1}{\sigma}\nabla.[(\nu+\tilde{\nu})\nabla\tilde{\nu} + c_{b2}(\nabla\tilde{\nu})^2]}_{\text{diffusion}} \tag{2}$$

Substituting 1 into 2 results in a RANS-type eddy-viscosity near the wall. The model length-scale reduces to $\tilde{d} = C_{DES}\Delta$ in the outer-layer, however. This leads to 3 which is the basis for Smagorisnky-type subgrid scale models.

$$\text{production} \approx \text{destruction} \quad \Longrightarrow \quad \tilde{\nu} \propto S\Delta^2 \tag{3}$$

Here, $S = (2S_{ij}S_{ij})^{\frac{1}{2}}$ where S_{ij} is the strain rate tensor. DES model suffers from the problem associated with the "grey area" when applied to attached, equilibrium flows [8, 9, 10]. Besides, it may lead to premature separation due to modeled-stress depletion when applied to a grid which is neither close to LES mesh density nor coarse enough to be considered as a RANS grid. These are some of the prevalent consequences of hybrid RANS/LES methods and notable effort has been spent so far to completely remove them or reduce their effect.

Piomelli *et al.* [10] noticed that the log-layer mismatch (LLM) can be completely removed from mean velocity profiles of turbulent channel flows if a correct value of stochastic forcing is applied in the inner-layer along with the hybrid calculation. Keating & Piomelli [11] proposed a dynamic method to estimate its magnitude with minimal user inputs. In addition to numerical instabilities, Davidson & Peng [12] observed that physical instabilities such as interaction of three-dimensional structures generated through a shear-layer may reduce LLM and lead to more accurate results. Furthermore, Spalart *et al.* [3] suggested to use a Delayed version of DES (DDES) which transfers the RANS/LES interface farther from the wall using a delay function, f_d, to avoid premature separation in cases with developing boundary layers or when local grid refinements are inevitable. Also, Shur *et al.* [4] introduced a new LES length-scale (l) that depends on the distance to the wall as well as the grid spacing, leading to a uniform distribution of l near the wall and far from the wall, and a linear transition in between. This is the key element to the improved DDES (IDDES) model. This new model is also claimed to resolve LLM.

In this study, the DES, DDES, and IDDES models are used for calculations of turbulent flow over a two-dimensional dune geometry at $Re_b = U_b H_b/\nu = 50000$. This case is neither a fully attached flow nor a massively separated flow and seems to be a good candidate to investigate the cost and accuracy of DES wall-models in such cases.

2 PROBLEM FORMULATION

The filtered conservation of mass and momentum are the governing equations as follows (4 and 5). The equations are discretized and solved on a structured curvilinear grid.

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \tag{4}$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} \tag{5}$$

Here, x_1, x_2, and x_3 (or x, y, and z, respectively) are the streamwise, wall-normal, and spanwise directions. To calculate the effect of unresolved scales on the flow, the dynamic procedure, proposed by Germano *et al.* [13] with Lagrangian averaging of Meneveau *et al.* [14] is used for the LES simulations in the present work. Furthermore, the SA-RANS is utilized as the base RANS model. The required modifications of the DES, DDES, and IDDES are then applied to the RANS equation for the purpose of hybrid calculations [2, 3, 4]. The computer code is second-order accurate in time and space which is well-validated through many publications [5, 15, 16].

The present test-case is a two-dimensional river dune, shown in figure 1. Here, $H_b = 3.5h$ is the average dune height. Also, U_b is the bulk velocity at the location with $H = H_b$. The simulations are performed at $Re_b = U_b H_b/\nu$ of 50000 (equivalent to $Re_\tau = u_\tau H_b/\nu = 2500$

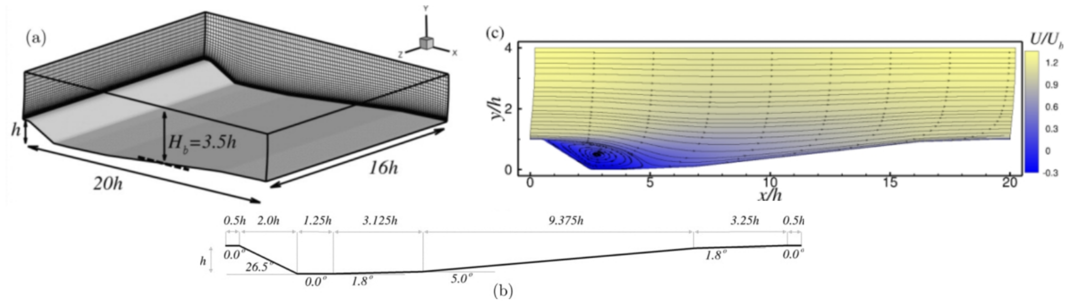

Figure 1: (a) Sketch of the computational domain; (b) bed geometry (extracted from [5] with permission); (c) mean flow streamlines; $Re_b = 50\,000$.

where u_τ is the friction velocity at the location with $H = H_b$), adequately high to analyze performance of the wall-layer models. Moreover, the boundary conditions are as follows: periodic in the x and z directions, no-slip at the wall, and zero shear stress (free-slip) on the top.

3 Results

3.1 Turbulence statistics

The RANS/LES interface (where $\langle uv \rangle_{\text{model}} = \langle uv \rangle_{\text{resolved}}$) and the wall shear stress at the dune bed are shown in figure 2. We could not specify any interface close to the wall over the recirculation region ($0.5 < x/h < 4.5$) and the majority of the total shear stress ($\langle uv \rangle_{\text{t}} = \langle uv \rangle_{\text{model}} + \langle uv \rangle_{\text{resolved}}$) is supported by the LES branch of the hybrid models (note that the possible intersection of $\langle uv \rangle_{\text{model}}$ and $\langle uv \rangle_{\text{resolved}}$ far from the wall is a result of the shear layer rather than the RANS-to-LES transition). Near the reattachment point, all the models predict a similar interface location due to the interaction of the three-dimensional structures, generated in the shear layer, with the interface ($4.5 < x/h < 7$). Farther downstream, the difference between the models becomes more visible; As expected, IDDES switches to LES closer to the bed while the DDES model shows a delayed transition as a result of a thicker RANS zone near the wall.

The aforementioned features have their own advantages and disadvantages when considering τ_w distribution over the dune bed. RANS models such as SA-RANS are not calibrated to account for geometry-induced scales, generated in the shear layer. As a result, all the hybrid models better predict τ_w compared to a pure RANS calculation on the same grid. Comparing different hybrid models, the reattachment point (where $\tau_w = 0$) is better predicted by the IDDES (separation is fixed because of the sharp edge on the lee

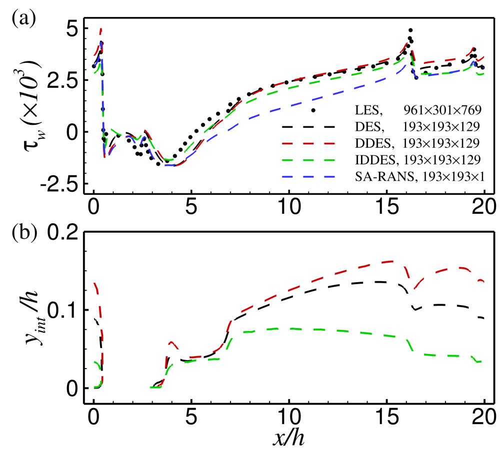

Figure 2: (a) Wall shear stress distribution over the dune bed; (b) interface distance to the dune bed; $Re_b = 50\,000$.

side). Moreover, over the recirculation region as well as the acceleration area near the next dune crest, IDDES results in a better consistency when τ_w is compared to the wall-resolved data. However, the two other models better estimate this quantity on the stoss side due to the quasi-equilibrium state of the flow over this region (the RANS models account for these effects).

The streamlines, and the mean separation and reattachment locations for each model are shown in Figure 3. The reattachment length is calculated based on zero mean wall shear stress criterion ($\tau_w = 0$). The mesh applied for these calculations resolves the step with an acceptable resolution ($\Delta s_i/h \approx 0.05$, this value is comparable to a coarse wall-resolved LES simulation at this Reynolds number with $\Delta s/h \approx 0.045$). The reattachment location, depends on the state of the boundary layer at separation, and is predicted quite differently using the three hybrid models. The reattachment length is over-predicted by 13%, 16%, and 10% us-

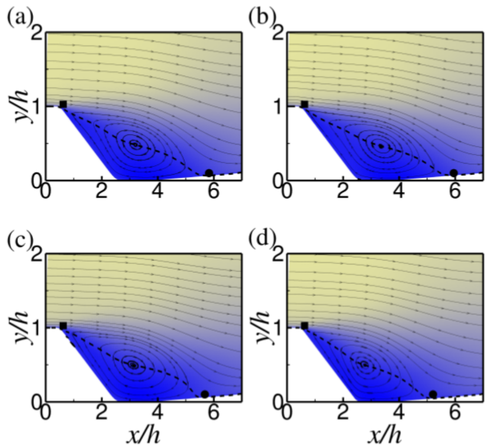

Figure 3: Recirculation bubble and average streamlines shown on the contours of the mean streamwise velocity (contour levels are the same as Figure 1 (c)); (a) DES; (b) DDES; (c) IDDES; (d) wall-resolved LES; the symbols, ■ and ● denote the mean separation and reattachment locations, respectively; - - - - -, zero-velocity line ($U = 0$); $Re_b = 50\,000$.

ing DES, DDES, and IDDES, respectively. Also, the shape of the recirculation bubble is compared qualitatively to the wall-resolved LES data. Defining the zero streamwise velocity line as our criterion for comparison, IDDES result has the best fit to the wall-resolved LES case over the two other hybrid models. Note that all of the hybrid simulations are carried out on the same mesh in order to remove all the effects associated with meshing technique and grid density.

Figure 4 (b) represents the total Reynolds shear stress in different streamwise locations. The IDDES model provides us with a more consistent data all over the domain (*e.g.* it always under-predicts $\langle uv\rangle_{\max}$ while the two other models' behavior depends on the streamwise location). Note that due to presence of the shear layer in this flow configuration, $\langle uv\rangle$ prediction still depends on the grid resolution in the LES core (regions within $0.5 < x/h < 7$) when IDDES is applied as the subgrid model.

The normalized mean velocity profile at $x/h = 18.5$ is shown in figure 4 (a). Here, $y^+ = u_\tau y/\nu$ and $U^+ = U/u_\tau$ (u_τ is the local friction velocity computed through each individual calculation). Considering the local RANS/LES interface for different models (shown by vertical lines), it is observable that the IDDES model's interface is located inside the buffer layer; a region with non-equilibrium effects. As a result a model with more LES content better estimates the velocity profile in this area (the over-prediction of

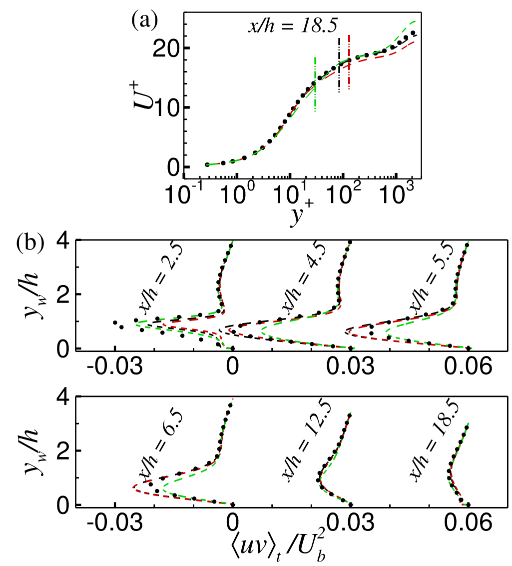

Figure 4: (a) the normalized velocity profile at $x/h = 18.5$; (b) Comparison of the total Reynolds shear stress at different streamwise locations; - - - - -, DES; - · - · -, DDES; - · · - · · -, IDDES; ● wall-resolved LES; $Re_b = 50\,000$.

the velocity in the free stream by IDDES is a result of under-prediction of local τ_w).

Figure 5 shows the resolved Turbulent Kinetic Energy (k) over the domain for different models.

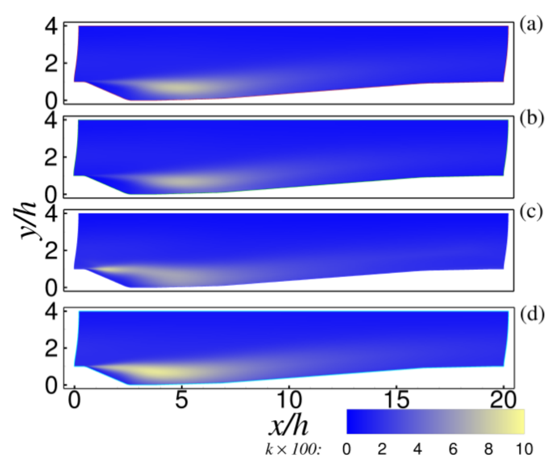

Figure 5: Contours of the resolved Turbulent Kinetic Energy (k); (a) DES; (b) DDES; (c) IDDES; (d) wall-resolved LES; $Re_b = 50\,000$.

All the hybrid methods estimate similar distribution of k in the shear layer and around the reattachment point, however, IDDES prediction is in better agreement with the wall-resolved LES data (under-predicted by a factor of 15%). We observed that the maximum k value is lower by 20% when comparing the DES and DDES results to the wall-resolved LES simulation, presumably because of the larger filter width. Over this region ($0.5 < x/h < 4.5$), although the LES branch of all the hybrid models is active (look at Figure 2 (b)), the effect of the new-born eddies becomes important and only those comparable to the filter width can be resolved while the subgrid scales carry large values of energy. As a result, LES becomes grid dependent no matter where the interface of RANS and LES regions is located.

3.2 Flow structures

The dominant geometry-induced structures in the flow over 2D dunes, such as those generated due to the instability of the shear layer at separation, are usually large enough to be resolved by a LES grid in the outer layer. As a result, neither the grid resolution, especially in wall-parallel directions, nor the wall-model itself applied to the near-wall area, should not affect these large scales. As observed in the previous section, the inner-outer layer interactions due to generation of these large scales can lead to improvements in prediction of the mean quantities using hybrid techniques.

Contours of the streamwise velocity fluctuations ($u^{'}$) are shown in Figure 6. Extraction of the data is performed in a plane parallel to the bed at $y_w/h = 0.04$ where y_w is the normal distance to the wall. At this distance, the DES and DDES models are still in the RANS mode while IDDES is in the RANS-to-LES transition over majority of the streamwise locations (all the methods result in a similar pattern for $u^{'}$ far from the wall).

Flow over 2D river dunes includes some important physics. The reattachment location fluctuates around a mean value, as a result of 3D, spanwise-oriented eddies. Farther downstream, acceleration of the flow on the stoss side forms large streamwise-oriented structures; their signature is observable on the bed as the stripes of high-speed streaks alternating with the low-speed ones.

Compared to the wall-resolved LES calculation at this Reynolds number, DES and DDES models show similar behavior all over the domain for $y_w/h = 0.04$, Figure 6 (a, b). Over the reattachment point, the footprint of 3D spanwise-oriented eddies, generated through the shear layer, is clearly observable and is in agreement with the reference wall-resolved calculation. These geometry-induced structures provide more entrainment at the RANS/LES interface and move it toward the wall. As can be seen in Figure 2 (b), DES and DDES predict similar transition location around the recirculation region despite of quite different interface location on the stoss side. Moreover, the reattachment point is also fluctuating similar to that of the wall-resolved LES case. On the stoss side, the size of the high-speed, low-speed stripes is over-predicted compared to the reference data. This is the consequence of a smooth RANS signal near the wall that is dictated by the simulation technique rather than the flow physics. Compared to a fully-developed channel flow calculation using the same models, a better entrainment between the RANS and LES area is achieved at this wall distance which is generally because of the streamwise vortices. These structures act as a mechanism to exchange momentum between the inner and outer-layer.

Subfigure (c) in Figure 6 shows the IDDES velocity fluctuations at the same wall distance. Generally, a better agreement is observed both in the recirculation region and on the stoss side of the dune. The reattachment point is fluctuating, analogous to the wall-resolved LES case and despite of a similar interface location around the reattachment point, the IDDES model resolves more scales compared to the two other wall-layers. Besides, on the attached flow region, IDDES estimates a more realistic distribution of the low-speed, high-speed stripes (regarding their size) due to its thinner RANS zone near the wall.

Omidyeganeh & Piomelli [5] visualized the turbulence structures of the flow over 2D river dunes at Reynolds of 18 900 using $640 \times 180 \times 640$ grid points in x, y, and z directions, respectively. This mesh was fine enough to make it possible to observe all the types of structures present in this physic: (i) rollers are generated at the crest due to the Kelvin-Helmholtz instability; (ii) horseshoe-like structures generated along the shear layer that either convect downstream or rise up to the surface; (iii) kolk vortices around the reattachment point; and (iv) large streamwise-oriented vortices on the stoss side. These outer-layer structures are mainly generated through the shear layer and are Reynolds number independent, subsequently, they must be also observed at all the higher Reynolds numbers.

Comparison of the outer-layer structures at Reynolds of 50 000, predicted by different wall-layer models as well as the wall-resolved LES calculation, is shown in Figure 7. We did not observe the kolk vortices at this Reynolds number using $p^{'} = -0.02$ (the kolk vortices are strongly rotating structures, similar to tornadoes, generated in the regions of high velocity gradients) using any of the simulations techniques. They

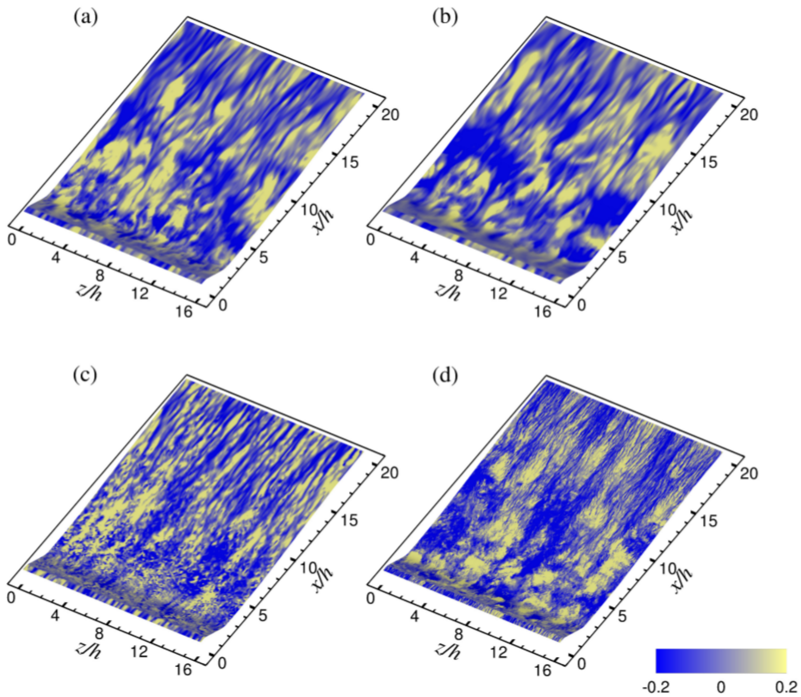

Figure 6: Contours of the streamwise velocity fluctuations on a plane parallel to the wall at $y_w/h = 0.04$; (a) DES; (b) DDES; (c) IDDES; (d) wall-resolved LES; $Re_b = 50\,000$.

are observed for smaller p', however. Note that the wall-resolved LES calculation for this study utilizes a mesh size of $961 \times 301 \times 769$ which is comparable to the intermediate simulation of Omidyeganeh & Piomelli [5] considering the grid resolution. We estimate that in order to finely resolve and be able to visualize all the structures, at least we would need up to $1\,500 \times 450 \times 1\,500$ grid points in the streamwise, wall-normal, and spanwise directions, respectively. However, the other three types of structures are usually large in size and can be resolved using a coarse grid. To do so, we extracted the instantaneous data for a duration equal to two flow-through times ($t^* = 40$ where $t^* = TU_b/h$) for each simulation technique. We successfully identified the horseshoe-like structures, rollers, and the streamwise vortices for each calculation.

A qualitative comparison of the DES and DDES simulations reveals that they have similar behavior in the recirculation region (a similar size of the resolved structures), nonetheless on the stoss side, as the flow convects downstream, DDES model predicts larger streamwise vortices which is also observable in Figure 6 where their signature on the bed is clearly seen as larger stripes of high-speed, low-speed streaks. In comparison with DES and DDES models, a larger range of scales is resolved using IDDES. This is attributed to a substantially thinner RANS area and a lower eddy-viscosity all over the domain predicted by the model, especially around the shear layer. However, some uncorrelated small-scale structures (also observed in Figure 6 (c)) are present in the isosurfaces of the pressure fluctuations all over the domain (Figure 7 (d)). We believe that this is because of insufficient turbulence dissipation near the reattachment point. Over this area, the large spanwise-oriented eddies touch the dune bed. Grid refinements can fix this issue.

4 Conclusion and Future Work

Wall-Modeled Large-Eddy Simulations (WMLES) were used for turbulent flow over two-dimensional river dunes using models of the Detached-Eddy Simulation family, including DES, DDES and IDDES methods. Simulations were carried out at $Re_b = 50000$ based on bulk velocity and average dune height. Wall-resolved LES calculations were also used to investigate the accuracy of the three wall-layers. Performing hybrid calculations using coarse meshes resulted in 98% and 97% of the mesh density and CPU time, respectively (the wall-resolved LES simulation required

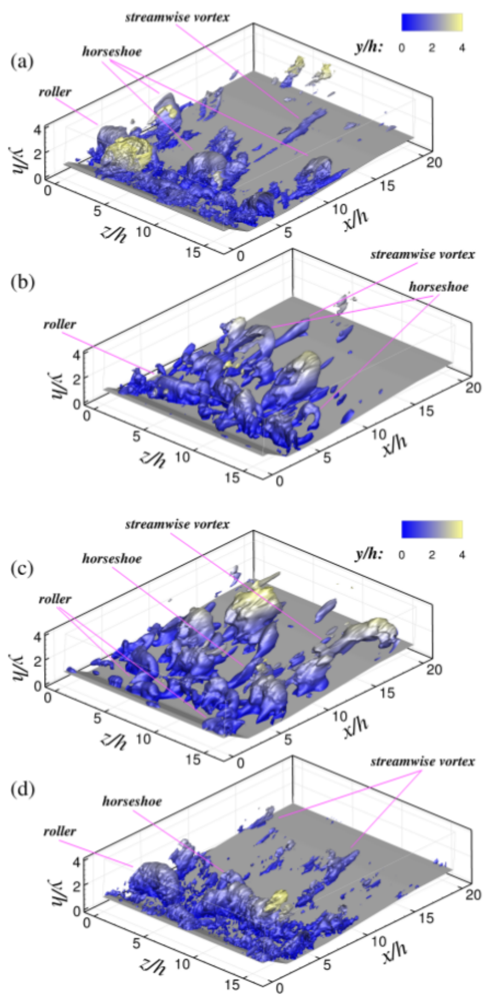

Figure 7: Instantaneous isosurfaces of the pressure fluctuations ($p' = -0.02$) colored with height; (a) LES; (b) DES; (c) DDES; (d) IDDES; $Re_b = 50000$.

276000 CPU hours for removing transient effects as well as statistical convergence).

Considering the instantaneous flow field, all the present wall-layers are able to resolve the coherent structures present in the flow over river dunes at high Reynolds numbers (wall-layers are generally designed for flow simulations at very high Reynolds numbers). However, IDDES may require a finer grid in the wall-parallel directions (x and z directions) compared to the two other models, DES and DDES, wherever a strong interaction is present between the large structures and the dune bed (*e.g.* over the reattachment location). This model switches to its LES content faster than the two other methods and its predicted eddy-viscosity is smaller than DES and DDES. Therefore, the dissipation of the structures interacting with the bed must be fed into the model by increasing the resolved dissipation using a finer grid. If the mesh density is not sufficiently fine, IDDES may over-estimates the normal Reynolds stresses near the bed.

Future work may involve the following directions: (i) revision of the DES-based methods to accelerate the eddy-generation process over the interface of the RANS and LES zones by imposing random noises in the solution of the subgrid eddy-viscosity; (ii) applying IDDES model to more realistic flow conditions for two-dimensional dunes in order to investigate the Reynolds number effects; (iii) studying sediment transport at high Reynolds numbers using present wall-models and eventually, simulating the movement of mobile sand beds.

All in all, the hybrid strategy seems to be a good candidate for turbulence simulation of flows with physical and geometry-induced instabilities (*e.g.* separation and flow over convex geometries).

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